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# Readers' Forum

*Brief discussion of previous investigations in the aerospace sciences and technical comments on papers published in the* AIAA Journal *are presented in this special department. Entries must be restricted to a maximum of 1000 words, or the equivalent of one* Journal *page including formulas and figures. A discussion will be published as quickly as possible after receipt of the manuscript. Neither the AIAA nor its editors are responsible for the opinions expressed by the correspondents. Authors will be invited to reply promptly.*

## Comment on "Compatibility of Measured and Predicted Vibration Modes in Model Improvement Studies"

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THE present writer offers some comments on the main issue of the subject paper[1] and suggests a possible improvement to treatment of that paper's final equation.

The subject paper has as its main point that there is generally an incompatibility between the analytical and experimental modal information for dynamic systems of interest. Therefore, correlation studies that seek to localize errors so as to improve the math model must either first 1) condense the analytical model or 2) expand the test data.

The present writer suggests that a third alternative is possible and at times superior to options 1 and 2 just cited; namely, to neither condense the analytical system degrees of freedom nor expand the test data base using the (perhaps less accurate) analytical model. This alternative simply compares available test data with its corresponding analytically determined values to determine the modeling corrections. In this way, neither the reduced analytical formulation nor the expanded modal test vectors become biased or contaminated. In addition, there is no requirement with this procedure that each test mode provide data for the same degrees of freedom. This latter condition could occur, for example, if a particular accelerometer gave reliable data for some modes but not for others.

A modal improvement/localization procedure that use this third alternative of modal comparison was first proposed by Collins et al.[2] and was later developed and implemented by Ojalvo et al.[3]

In this procedure, one uses a Taylor expansion of the less-detailed modal test data $\{\phi_T\}$ and corresponding truncated analytical data $\{\bar{\phi}_A\}$ to obtain

$$\{\phi_T\} = \{\bar{\phi}_A\} + [S]\{\Delta r\} + \{R\} \tag{1}$$

where

$$[S] = \left[\frac{\partial\{\bar{\phi}_A\}}{\partial\{r\}}\right] \tag{2}$$

where $\{\Delta r\}$ are the analytical model error localization or parameter corrections to improve correlation, and $\{R\}$ are the corresponding residual terms of the Taylor series plus experimental errors.

A least-squares minimization of the residuals in the weighted form

$$\{R\}^T[W]\{R\}$$

then leads to the system equations

$$[S]^T[W][S]\{\Delta r\} = [S]^T[W](\{\phi_T\} - \{\bar{\phi}_A\}) \tag{3}$$

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which may be solved for $\{\Delta r\}$ by ordinary decomposition of $[S]^T[W][S]$ if it is nonsingular or by epsilon decomposition[4] if it is singular.

The second point that the present writer addresses concerns the subject paper's Eq. (10), in which the matrix product $[\phi_c][\phi_c]^T$ is rank deficient. Taking the transpose of Eq. (10) in Ref. 1 leads to

$$[\phi_c][\phi_c]^T([\Delta K] + i[H]) = [\phi_c][\cdot\,\omega_c^2\,\cdot][\phi_c]^T[M_c] - [\phi_c][\phi_c]^T[K_a] \tag{4}$$

This equation may be solved for $([\Delta K] + i[H])$ via singular value decomposition (SVD).[5] The resulting solution, although not unique, will yield the "minimum norm" solution for $([\Delta K] + i[H])$, i.e., the solution for which $[K_c]$ is closest to $[K_a]$ and is often physically most justifiable.

The SVD solution for Eq. (4) is

$$([\Delta K] + i[H]) = X\Lambda^{-1}X^T(\phi_c\omega_c^2\phi_c^TM_c - \phi_c\phi_c^TK_a) \tag{5}$$

where $\Lambda$ are the nonzero eigenvalues of $[\phi_c]$ and $[\phi_c]^T$ and $X$ are its corresponding eigenvectors.

A final minor point is made here regarding the subject paper's Fig. 2, in which the square shaded areas represented in $[K_a]_{12}$ and $[K_a]_{21}$ should be interchanged.

### References

[1]He, J., and Ewins, D. J., "Compatibility of Measured and Predicted Vibration Modes in Model Improvement Studies," *AIAA Journal,* Vol. 29, No. 5, 1991, pp. 798–803.

[2]Collins, J. D., Hart, G. C., Hasselman, T. K., and Kennedy, B., "Statistical Identification of Structures," *AIAA Journal,* Vol. 12, No. 2, 1974, pp. 185–190.

[3]Ojalvo, I. U., Ting, T., and Pilon, D., "PAREDYM—A Parameter Refinement Computer Code for Structural Dynamic Models," *International Journal of Analytical and Experimental Modal Analysis,* Vol. 5, No. 1, 1989, pp. 43–49.

[4]Ojalvo, I. U., and Ting, T., "Interpretation and Improved Solution Approach for Ill-Conditioned Linear Equations," *AIAA Journal,* Vol. 28, No. 11, 1990, pp. 1976–1979.

[5]Maia, N. M. M., "An Introduction to the Singular Value Decomposition Technique (SVD)," *Proceedings of the 7th International Modal Analysis Conference,* Vol. 1, Union College, Schenectady, NY, and Society for Experimental Mechanics, Bethel, CT, 1990, pp. 335–339.

## Reply by the Authors to I. U. Ojalvo

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THE authors thank I. U. Ojalvo for his interest in and comments on their recent paper[1] and wish to affirm and expand on the points made in his Comment.

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The first remark is that there exists a third alternative to the two choices cited in Ref. 1; namely, a need to 1) reduce or condense the analytical model to the size (coordinate set) of the experimental model or 2) expand the experimental mode shapes to the full analytical model size.

The third alternative, discussed in the Comment, is simply to use those data that are available without seeking to make both analytical and experimental models be of the same order.

The authors agree that this is an alternative strategy that can be implemented not only in the sensitivity method outlined in the Comment but also in the error matrix method[3] and in the more recent response function method.[4] However, there exist a number of model improvement methods that *do* require expansion to the full size, and these are generally ones that incorporate a version of the orthogonality properties in their formulation, including the direct matrix update and eigendynamic constraint methods. For these, the third option is not really applicable.

The second remark refers to Eq. (10) of the original paper.[1] The authors acknowledge the possibility discussed in the Comment of implementing a singular value decomposition based solution for the unknown $[\Delta K]$ and $[H]$ matrices. Indeed, a very similar procedure has been explored in Ref. 4. However, the context of Eq. (10) in the paper is its potential for *locating* regions of error without the need for any matrix inversion at all. The point made in the paper is that the matrix expression on the right-hand side of Eq. (10), which can be readily computed, displays explicitly the error-affected coordinates (rows and columns) of the model. In this respect, the expression is offered as a convenient way to focus attention on the important areas of the model (error localization), a process that can greatly improve the subsequent updating task.

### References

[1]He, J., and Ewins, D. J., "Compatibility of Measured and Predicted Vibration Modes in Model Improvement Studies," *AIAA Journal,* Vol. 29, No. 5, 1991, pp. 798–803.

[2]Sidhu, J., and Ewins, D. J., "Correlation of Finite Element and Modal Test Studies of a Practical Structure," *Proceedings of the 2nd International Modal Analysis Conference,* Union College, Schenectady, NY, and Society for Experimental Mechanics, Bethel, CT, 1984, pp. 756–762.

[3]Lin, R.-M., and Ewins, D. J., "Model Updating Using FRF Data," *Proc. Int. Conf. Modal Analysis,* Leuven, Belgium, Sept. 1990.

[4]Leiven, N., and Ewins, D. J., "Error Location and Updating of Finite Elements Models Using Singular Value Decomposition," *Proceedings of the 8th International Modal Analysis Conference,* Union College, Schenectady, NY, and Society for Experimental Mechanics, Bethel, CT, 1990, pp. 768–773.

# Errata

## Exploratory Design Studies of Actively Controlled Wings Using Integrated Multidisciplinary Synthesis

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DURING final corrections to this article, E. Livne's name was inadvertently misspelled. We regret this error.